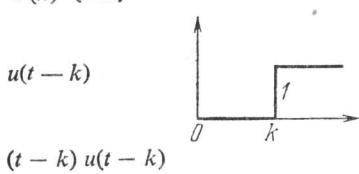


29.3. ТАБЛИЦА ПРЕОБРАЗОВАНИЙ ЛАПЛАСА

	$f(s)$	$F(t)$
29.3.1.	$\frac{1}{s}$	1
29.3.2.	$\frac{1}{s^2}$	t
29.3.3.	$\frac{1}{s^n}$ ($n = 1, 2, 3, \dots$)	$\frac{t^{n-1}}{(n-1)!}$
29.3.4.	$\frac{1}{\sqrt{s}}$	$\frac{1}{\sqrt{\pi t}}$
29.3.5.	$s^{-3/2}$	$2\sqrt{t/\pi}$
29.3.6.	$s^{-(n+1/2)}$ ($n = 1, 2, 3, \dots$)	$\frac{2^n t^{n-1/2}}{1 \cdot 3 \cdot 5 \dots (2n-1) \sqrt{\pi}}$
29.3.7.	$\frac{\Gamma(k)}{s^k}$ ($k > 0$)	t^{k-1}
29.3.8.	$\frac{1}{s+a}$	e^{-at}
29.3.9.	$\frac{1}{(s+a)^2}$	te^{-at}
29.3.10.	$\frac{1}{(s+a)^n}$ ($n = 1, 2, 3, \dots$)	$\frac{t^{n-1}e^{-at}}{(n-1)!}$
29.3.11.	$\frac{\Gamma(k)}{(s-a)^k}$ ($k > 0$)	$t^{k-1}e^{-at}$
29.3.12.	$\frac{1}{(s+a)(s+b)}$ ($a \neq b$)	$\frac{e^{-at} - e^{-bt}}{b-a}$
29.3.13.	$\frac{s}{(s+a)(s+b)}$ ($a \neq b$)	$\frac{ae^{-at} - be^{-bt}}{a-b}$
29.3.14.	$\frac{1}{(s+a)(s+b)(s+c)}$	$-\frac{(b-c)e^{-at} + (c-a)e^{-bt} + (a-b)e^{-ct}}{(a-b)(b-c)(c-a)}$
(a, b, c — различные постоянные)		
29.3.15.	$\frac{1}{s^2 + a^2}$	$\frac{1}{a} \sin at$
29.3.16.	$\frac{s}{s^2 + a^2}$	$\cos at$
29.3.17.	$\frac{1}{s^2 - a^2}$	$\frac{1}{a} \operatorname{sh} at$
29.3.18.	$\frac{s}{s^2 - a^2}$	$\operatorname{ch} at$
29.3.19.	$\frac{1}{s(s^2 + a^2)}$	$\frac{1}{a^2} (1 - \cos at)$

$f(s)$	$F(t)$
29.3.20. $\frac{1}{s^2(s^2 + a^2)}$	$\frac{1}{a^3} (at - \sin at)$
29.3.21. $\frac{1}{(s^2 + a^2)^2}$	$\frac{1}{2a^5} (\sin at - at \cos at)$
29.3.22. $\frac{s}{(s^2 + a^2)^2}$	$\frac{t}{2a} \sin at$
29.3.23. $\frac{s^2}{(s^2 + a^2)^2}$	$\frac{1}{2a} (\sin at + at \cos at)$
29.3.24. $\frac{s^2 - a^2}{(s^2 + a^2)^2}$	$t \cos at$
29.3.25. $\frac{s}{(s^2 + a^2)(s^2 + b^2)} \quad (a^2 \neq b^2)$	$\frac{\cos at - \cos bt}{b^2 - a^2}$
29.3.26. $\frac{1}{(s + a)^2 + b^2}$	$\frac{1}{b} e^{-at} \sin bt$
29.3.27. $\frac{s + a}{(s + a)^2 + b^2}$	$e^{-at} \cos bt$
29.3.28. $\frac{3a^2}{s^3 + a^3}$	$e^{-at} - e^{at/2} \left(\cos \frac{at\sqrt{3}}{2} - \sqrt{3} \sin \frac{at\sqrt{3}}{2} \right)$
29.3.29. $\frac{4a^3}{s^4 + 4a^4}$	$\sin at \operatorname{ch} at - \cos at \operatorname{sh} at$
29.3.30. $\frac{s}{s^4 + 4a^4}$	$\frac{1}{2a^2} \sin at \operatorname{sh} at$
29.3.31. $\frac{1}{s^4 - a^4}$	$\frac{1}{2a^3} (\operatorname{sh} at - \sin at)$
29.3.32. $\frac{s}{s^4 - a^4}$	$\frac{1}{2a^2} (\operatorname{ch} at - \cos at)$
29.3.33. $\frac{8a^3s^2}{(s^2 + a^2)^3}$	$(1 + a^2 t^2) \sin at - at \cos at$
29.3.34. $\frac{1}{s} \left(\frac{s-1}{s} \right)^n$	$L_n(t)$
29.3.35. $\frac{s}{(s+a)^{3/2}}$	$\frac{1}{\sqrt{\pi t}} e^{-at} (1 - 2at)$
29.3.36. $\sqrt{s+a} - \sqrt{s+b}$	$\frac{1}{2\sqrt{\pi t^3}} (e^{-bt} - e^{-at})$
29.3.37. $\frac{1}{\sqrt{s+a}}$	$\frac{1}{\sqrt{\pi t}} - ae^{a^2 t} \operatorname{erfc} a \sqrt{t}$
29.3.38. $\frac{\sqrt{s}}{s-a^2}$	$\frac{1}{\sqrt{\pi t}} + ae^{a^2 t} \operatorname{erf} a \sqrt{t}$
29.3.39. $\frac{\sqrt{s}}{s+a^2}$	$\frac{1}{\sqrt{\pi t}} - \frac{2a}{\sqrt{\pi}} e^{-a^2 t} \int_0^{a\sqrt{t}} e^{\lambda^2} d\lambda$
29.3.40. $\frac{1}{\sqrt{s}(s-a^2)}$	$\frac{1}{a} e^{at} \operatorname{erf} a \sqrt{t}$
29.3.41. $\frac{1}{\sqrt{s}(s+a^2)}$	$\frac{2}{a\sqrt{\pi}} e^{-a^2 t} \int_0^{a\sqrt{t}} e^{\lambda^2} d\lambda$

$f(s)$	$F(t)$
29.3.42. $\frac{b^2 - a^2}{(s - a^2)(b + \sqrt{s})}$	$e^{at} [b - a \operatorname{erf} a \sqrt{t}] - b e^{bt} \operatorname{erfc} b \sqrt{t}$
29.3.43. $\frac{1}{\sqrt{s}(\sqrt{s} + a)}$	$e^{at} \operatorname{erfc} a \sqrt{t}$
29.3.44. $\frac{1}{(s + a)\sqrt{s} + b}$	$\frac{1}{\sqrt{b-a}} e^{-at} \operatorname{erf} (\sqrt{b-a} \sqrt{t})$
29.3.45. $\frac{b^2 - a^2}{\sqrt{s}(s - a^2)(\sqrt{s} + b)}$	$e^{at} \left[\frac{b}{a} \operatorname{erf} (a \sqrt{t}) - 1 \right] + e^{bt} \operatorname{erfc} b \sqrt{t}$
29.3.46. $\frac{(1-s)^n}{s^{n+1/2}}$	$\frac{n!}{(2n)! \sqrt{\pi t}} H_{2n}(\sqrt{t})$
29.3.47. $\frac{(1-s)^n}{s^{n+1/2}}$	$\frac{n!}{(2n+1)! \sqrt{\pi}} H_{2n+1}(\sqrt{t})$
29.3.48. $\frac{\sqrt{s+2a}}{\sqrt[4]{s}} - 1$	$a e^{-at} [I_1(at) + I_0(at)]$
29.3.49. $\frac{1}{\sqrt{s+a}\sqrt{s+b}}$	$e^{-\frac{1}{2}(a+b)t} I_0 \left(\frac{a-b}{2} t \right)$
29.3.50. $\frac{\Gamma(k)}{(s+a)^k(s+b)^k} \quad (k > 0)$	$\sqrt{\pi} \left(\frac{t}{a-b} \right)^{k-1/2} e^{-\frac{1}{2}(a+b)t} I_{k-1/2} \left(\frac{a-b}{2} t \right)$
29.3.51. $\frac{1}{(s+a)^{1/2}(s+b)^{3/2}}$	$t e^{-\frac{1}{2}(a+b)t} \left[I_0 \left(\frac{a-b}{2} t \right) + I_1 \left(\frac{a-b}{2} t \right) \right]$
29.3.52. $\frac{\sqrt{s+2a} - \sqrt{s}}{\sqrt{s+2a} + \sqrt{s}}$	$\frac{1}{t} e^{-at} I_1(at)$
29.3.53. $\frac{(a-b)^k}{(\sqrt{s+a} + \sqrt{s+b})^{2k}} \quad (k > 0)$	$\frac{k}{t} e^{-\frac{1}{2}(a+b)t} I_k \left(\frac{a-b}{2} t \right)$
29.3.54. $\frac{(\sqrt{s+a} + \sqrt{s})^{-2v}}{\sqrt{s}\sqrt{s+a}} \quad (v > -1)$	$\frac{1}{a^v} e^{-\frac{1}{2}at} I_v \left(\frac{1}{2} at \right)$
29.3.55. $\frac{1}{\sqrt{s^2 + a^2}}$	$J_0(at)$
29.3.56. $\frac{(\sqrt{s^2 + a^2} - s)^v}{\sqrt{s^2 + a^2}} \quad (v > -1)$	$a^v J_v(at)$
29.3.57. $\frac{1}{(s^2 + a^2)^k} \quad (k > 0)$	$\frac{\sqrt{\pi}}{\Gamma(k)} \left(\frac{t}{2a} \right)^{k-1/2} J_{k-1/2}(at)$
29.3.58. $(\sqrt{s^2 + a^2} - s)^k \quad (k > 0)$	$\frac{ka^k}{t} J_k(at)$
29.3.59. $\frac{(s - \sqrt{s^2 - a^2})^v}{\sqrt{s^2 - a^2}} \quad (v > -1)$	$a^v I_v(at)$
29.3.60. $\frac{1}{(s^2 - a^2)^k} \quad (k > 0)$	$\frac{\sqrt{\pi}}{\Gamma(k)} \left(\frac{t}{2a} \right)^{k-1/2} I_{k-1/2}(at)$
29.3.61. $\frac{1}{s} e^{-ks}$	$u(t-k)$
29.3.62. $\frac{1}{s^2} e^{-ks}$	$(t-k) u(t-k)$

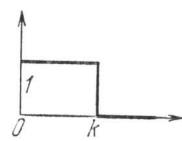


$f(s)$

$$29.3.63. \frac{1}{s^\mu} e^{-ks} \quad (\mu > 0)$$

 $F(t)$

$$\frac{(t-k)^{\mu-1}}{\Gamma(\mu)} u(t-k)$$

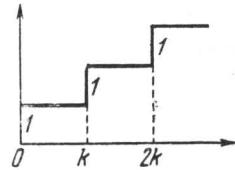


$$29.3.64. \frac{1 - e^{-ks}}{s}$$

$$u(t) - u(t-k)$$

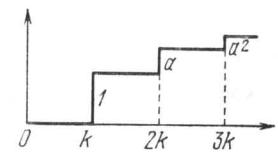
$$29.3.65. \frac{1}{s(1 - e^{-ks})} = \frac{1 + \coth \frac{1}{2} ks}{2s}$$

$$\sum_{n=0}^{\infty} u(t-nk)$$



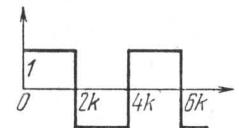
$$29.3.66. \frac{1}{s(e^{ks} - a)}$$

$$\sum_{n=1}^{\infty} a^{n-1} u(t-nk)$$



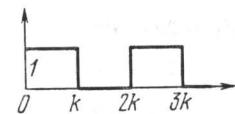
$$29.3.67. \frac{1}{s} \operatorname{th} ks$$

$$u(t) + 2 \sum_{n=1}^{\infty} (-1)^n u(t-2nk)$$



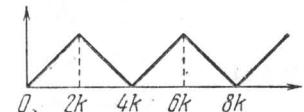
$$29.3.68. \frac{1}{s(1 + e^{-ks})}$$

$$\sum_{n=0}^{\infty} (-1)^n u(t-nk)$$



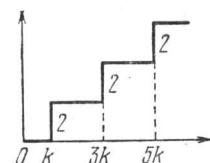
$$29.3.69. \frac{1}{s^2} \operatorname{ta} ks$$

$$tu(t) + 2 \sum_{n=1}^{\infty} (-1)^n \times \\ \times (t-2nk) u(t-2nk)$$



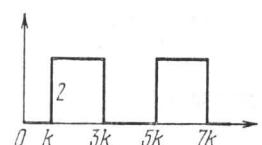
$$29.3.70. \frac{1}{s \operatorname{sh} ks}$$

$$2 \sum_{n=0}^{\infty} u[t-(2n+1)k]$$



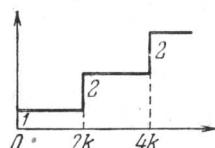
$$29.3.71. \frac{1}{s \operatorname{ch} ks}$$

$$2 \sum_{n=0}^{\infty} (-1)^n u[t-(2n+1)k]$$

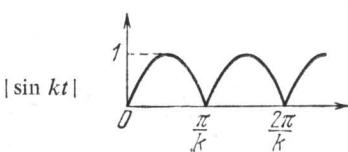


$$29.3.72. \frac{1}{s} \operatorname{cth} ks$$

$$u(t) + 2 \sum_{n=1}^{\infty} u(t-2nk)$$



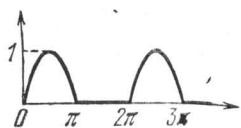
$$29.3.73. \frac{k}{s^2 + k^2} \operatorname{cth} \frac{\pi s}{2k}$$



$f(s)$ $F(t)$

$$29.3.74. \frac{1}{(s^2 + 1)(1 - e^{-\pi s})}$$

$$\sum_{n=0}^{\infty} (-1)^n u(t - n\pi) \sin t$$



$$29.3.75. \frac{1}{s} e^{-k/s}$$

$$J_0(2\sqrt{kt})$$

$$29.3.76. \frac{1}{\sqrt{s}} e^{-k/s}$$

$$\frac{1}{\sqrt{\pi t}} \cos 2\sqrt{kt}$$

$$29.3.77. \frac{1}{\sqrt{s}} e^{k/s}$$

$$\frac{1}{\sqrt{\pi t}} \operatorname{ch} 2\sqrt{kt}$$

$$29.3.78. \frac{1}{s^{3/2}} e^{-k/s}$$

$$\frac{1}{\sqrt{\pi k}} \sin 2\sqrt{kt}$$

$$29.3.79. \frac{1}{s^{\mu/2}} e^{k/s}$$

$$\frac{1}{\sqrt{\pi k}} \operatorname{sh} 2\sqrt{kt}$$

$$29.3.80. \frac{1}{s^\mu} e^{-k/s} \quad (\mu > 0)$$

$$\left(\frac{t}{k}\right)^{(\mu-1)/2} J_{\mu-1}(2\sqrt{kt})$$

$$29.3.81. \frac{1}{s^\mu} e^{k/s} \quad (\mu > 0)$$

$$\left(\frac{t}{k}\right)^{(\mu-1)/2} I_{\mu-1}(2\sqrt{kt})$$

$$29.3.82. e^{-k\sqrt{s}} \quad (k > 0)$$

$$\frac{k}{2\sqrt{\pi t^3}} \exp\left(-\frac{k^2}{4t}\right)$$

$$29.3.83. \frac{1}{s} e^{-k\sqrt{s}} \quad (k \geq 0)$$

$$\operatorname{erfc} \frac{k}{2\sqrt{t}}$$

$$29.3.84. \frac{1}{\sqrt{s}} e^{-k\sqrt{s}} \quad (k \geq 0)$$

$$\frac{1}{\sqrt{\pi t}} \exp\left(-\frac{k^2}{4t}\right)$$

$$29.3.85. \frac{1}{s^{3/2}} e^{-k\sqrt{s}} \quad (k \geq 0)$$

$$2\sqrt{\frac{t}{\pi}} \exp\left(-\frac{k^2}{4t}\right) - k \operatorname{erfc} \frac{k}{2\sqrt{t}} = 2\sqrt{t} i \operatorname{erfc} \frac{k}{2\sqrt{t}}$$

$$29.3.86. \frac{1}{s^{1+n/2}} e^{-k\sqrt{s}} \quad (n = 0, 1, 2, \dots, k \geq 0)$$

$$(4t)^{n/2} i^n \operatorname{erfc} \frac{k}{2\sqrt{t}}$$

$$29.3.87. s^{(n-1)/2} e^{-k\sqrt{s}} \quad (n = 0, 1, 2, \dots; k > 0)$$

$$\frac{\exp\left(-\frac{k^2}{4t}\right)}{2^n \sqrt{\pi t^{n+1}}} H_n\left(\frac{k}{2\sqrt{t}}\right)$$

$$29.3.88. \frac{e^{-k\sqrt{s}}}{a + \sqrt{s}} \quad (k \geq 0)$$

$$\frac{1}{\sqrt{\pi t}} \exp\left(-\frac{k^2}{4t}\right) - a e^{ak} e^{a^2 t} \operatorname{erfc}\left(a\sqrt{t} + \frac{k}{2\sqrt{t}}\right)$$

$$29.3.89. \frac{a e^{-k\sqrt{s}}}{s(a + \sqrt{s})} \quad (k \geq 0)$$

$$-e^{ak} e^{a^2 t} \operatorname{erfc}\left(a\sqrt{t} + \frac{k}{2\sqrt{t}}\right) + \operatorname{erfc} \frac{k}{2\sqrt{t}}$$

$$29.3.90. \frac{e^{-k\sqrt{s}}}{\sqrt{s}(a + \sqrt{s})} \quad (k \geq 0)$$

$$e^{ak} e^{a^2 t} \operatorname{erfc}\left(a\sqrt{t} + \frac{k}{2\sqrt{t}}\right)$$

$$29.3.91. \frac{e^{-k\sqrt{s(s+a)}}}{\sqrt{s(s+a)}} \quad (k \geq 0)$$

$$e^{-at/2} I_0\left(\frac{1}{2} a \sqrt{t^2 - k^2}\right) u(t - k)$$

$$29.3.92. \frac{e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}} \quad (k \geq 0)$$

$$J_0(a \sqrt{t^2 - k^2}) u(t - k)$$

$f(s)$	$F(t)$
29.3.93. $\frac{e^{-k\sqrt{s^2-a^2}}}{\sqrt{s^2-a^2}} \quad (k \geq 0)$	$I_0(a\sqrt{t^2-k^2}) u(t-k)$
29.3.94. $\frac{e^{-k(\sqrt{s^2+a^2}-s)}}{\sqrt{s^2+a^2}} \quad (k \geq 0)$	$J_0(a\sqrt{t^2+2kt})$
29.3.95. $e^{-ks} - e^{-k\sqrt{s^2+a^2}} \quad (k > 0)$	
29.3.96. $e^{-k\sqrt{s^2-a^2}} - e^{-ks} \quad (k > 0)$	
29.3.97. $\frac{a^v e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}(\sqrt{s^2+a^2+s})^v} \quad (v > -1, k \geq 0)$	$\frac{ak}{\sqrt{t^2-k^2}} J_1(a\sqrt{t^2-k^2}) u(t-k)$
29.3.98. $\frac{1}{s} \ln s$	$\frac{ak}{\sqrt{t^2-k^2}} I_1(a\sqrt{t^2-k^2}) u(t-k)$
29.3.99. $\frac{1}{s^k} \ln s \quad (k > 0)$	$\left(\frac{t-k}{t+k} \right)^{v/2} J_v(a\sqrt{t^2-k^2}) u(t-k)$
29.3.100. $\frac{\ln s}{s-a} \quad (a > 0)$	$-\gamma - \ln t \quad (\gamma = 0.57721 56649 \dots \text{ — постоянная Эйлера})$
29.3.101. $\frac{\ln s}{s^2+1}$	$\frac{t^{k-1}}{\Gamma(k)} [\psi(k) - \ln t]$
29.3.102. $\frac{s \ln s}{s^2+1}$	$e^{at} [\ln a + E_1(at)]$
29.3.103. $\frac{1}{s} \ln(1+ks) \quad (k > 0)$	$\cos t \operatorname{Si}(t) - \sin t \operatorname{Ci}(t)$
29.3.104. $\ln \frac{s+a}{s+b}$	$-\sin t \operatorname{Si}(t) - \cos t \operatorname{Ci}(t)$
29.3.105. $\frac{1}{s} \ln(1+k^2 s^2) \quad (k > 0)$	$E_1\left(\frac{t}{k}\right)$
29.3.106. $\frac{1}{s} \ln(s^2+a^2) \quad (a > 0)$	$\frac{1}{t} (e^{-bt} - e^{-at})$
29.3.107. $\frac{1}{s^2} \ln(s^2+a^2) \quad (a > 0)$	$-2 \operatorname{Ci}\left(\frac{t}{k}\right)$
29.3.108. $\ln \frac{s^2+a^2}{s^2}$	$2 \ln a - 2 \operatorname{Ci}(at)$
29.3.109. $\ln \frac{s^2-a^2}{s^2}$	$\frac{2}{a} [at \ln a + \sin at - at \operatorname{Ci}(at)]$
29.3.110. $\operatorname{arctg} \frac{k}{s}$	$\frac{2}{t} (1 - \cos at)$
29.3.111. $\frac{1}{s} \operatorname{arctg} \frac{k}{s}$	$\frac{2}{t} (1 - \operatorname{ch} at)$
29.3.112. $e^{ks} \operatorname{erfc} ks \quad (k > 0)$	$\frac{1}{t} \sin kt$
29.3.113. $\frac{1}{s} e^{ks} \operatorname{erfc} ks \quad (k > 0)$	$\operatorname{Si}(kt)$
29.3.114. $e^{ks} \operatorname{erfc} \sqrt{ks} \quad (k > 0)$	$\frac{1}{k\sqrt{\pi}} \exp\left(-\frac{t^2}{4k^2}\right)$
	$\operatorname{erf} \frac{t}{2k}$
	$\frac{\sqrt{k}}{\pi\sqrt{t(t+k)}}$

$f(s)$	$F(t)$
29.3.115. $\frac{1}{\sqrt{s}} \operatorname{erfc} \sqrt{ks}$ ($k \geq 0$)	$\frac{1}{\sqrt{\pi t}} u(t - k)$
29.3.116. $\frac{1}{\sqrt{s}} e^{ks} \operatorname{erfc} \sqrt{ks}$ ($k \geq 0$)	$\frac{1}{\sqrt{\pi(t + k)}}$
29.3.117. $\operatorname{erf} \frac{k}{\sqrt{s}}$	$\frac{1}{\pi t} \sin 2k \sqrt{t}$
29.3.118. $\frac{1}{\sqrt{s}} e^{k^2/s} \operatorname{erfc} \frac{k}{\sqrt{s}}$	$\frac{1}{\sqrt{\pi t}} e^{-2k \sqrt{t}}$
29.3.119. $K_0(ks)$ ($k > 0$)	$\frac{1}{\sqrt{t^2 - k^2}} u(t - k)$
29.3.120. $K_0(k \sqrt{s})$ ($k > 0$)	$\frac{1}{2t} \exp \left(-\frac{k^2}{4t} \right)$
29.3.121. $\frac{1}{s} e^{ks} K_1(ks)$ ($k > 0$)	$\frac{1}{k} \sqrt{t(t + 2k)}$
29.3.122. $\frac{1}{\sqrt{s}} K_1(k \sqrt{s})$ ($k > 0$)	$\frac{1}{k} \exp \left(-\frac{k^2}{4t} \right)$
29.3.123. $\frac{1}{\sqrt{s}} e^{k/s} K_0 \left(\frac{k}{s} \right)$ ($k > 0$)	$\frac{2}{\sqrt{\pi t}} K_0(2\sqrt{2kt})$
29.3.124. $\pi e^{-ks} I_0(ks)$ ($k > 0$)	$\frac{1}{\sqrt{t(2k - t)}} [u(t) - u(t - 2k)]$
29.3.125. $e^{-ks} I_1(ks)$ ($k > 0$)	$\frac{k - t}{\pi k \sqrt{t(2k - t)}} [u(t) - u(t - 2k)]$
29.3.126. $e^{as} E_1(as)$ ($a > 0$)	$\frac{1}{t + a}$
29.3.127. $\frac{1}{a} - se^{as} E_1(as)$ ($a > 0$)	$\frac{1}{(t + a)^2}$
29.3.128. $a^{1-n} e^{as} E_n(as)$ ($a > 0; n = 0, 1, 2, \dots$)	$\frac{1}{(t + a)^n}$
29.3.129. $\left[\frac{\pi}{2} - \operatorname{Si}(s) \right] \cos s + \operatorname{Ci}(s) \sin s$	$\frac{1}{t^2 + 1}$